Forecasting Earnings Variance: Quantile-Based Vs. Residuals-Based Variance Proxies*

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Abstract

Information about future moments of earnings, i.e. earnings uncertainty, are relevant to any agent whose wealth is dependent on earnings. The aim of this study is to analyze different forecasting approaches for the variance of future earnings, compare the respective forecast accuracy and test whether the forecasted information are relevant to equity or debt markets. The results, in line with former research, indicate that quantile-based variance proxies outperform resiudal-based variance proxies in industry-level tests. However, a residual-based variance proxy outperforms the quantile-based variance proxies in terms of forecast accuracy in firm-level tests. This study finds that the opposing performance outcome appears to be driven by the proneness of quantile-based variance proxies to produce comparably large forecasts in the extreme percentiles whose influence is masked in the industry-, but not in the firm-level test. Further, this study finds the book-value of equity to be an important predictor for the future earnings variance overlooked in previous studies. Finally, it confirms findings from former studies that equity prices are increasing in the variance of future earnings indicating a mechanism similar to the idea of options pricing theory, whereas there is no significant relationship between bond ratings and the variance of future earnings.

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JEL Classification: G17, G32, M41

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1 Introduction

Extensive research has been conducted on forecasting future mean earnings, i.e. the first moment of future earnings (e.g., Hou, Van Dijk and Zhang (2012), Li and Mohanram (2014), Ohlson and Kim (2015), Evans, Njoroge and Yong (2017), Cao and You (2020), Tian, Yim and Newton (2021) and Hendriock (2022)). However, information about the higher moments of future earnings are also important in various economic settings and to a range of economic agents, although methodological suggestions are sparse in this comparably novel stem of research.

In general, earnings uncertainty, i.e. the entirety of higher moments of future earnings, is relevant to any agent whose wealth is either directly or indirectly dependent on earnings (Chang, Monahan, Ouazad and Vasvari (2021), hereafter CH). More specifically, CH as well as Konstantinidi and Pope (2016), hereafter KP, show that both the value of debt and equity are related to higher moments of future earnings. Additionally, former research led to the conclusion that risk in earnings affects future growth persistence which then influences the predictability of earnings and subsequently a firm's valuation (e.g., Dichev and Tang (2009) and Penman and Zhang (2002)). These results, so argue Dichev and Tang (2009), fall in line with the findings by Graham, Harvey and Rajgopal (2005) who show that executives believe earnings predictability to be negatively related to earnings volatility. Further, research has shown that equity value as well as equity prices are a function of, inter alia, the higher moments of future earnings (e.g. Merton (1987), Johnson (2004), Brunnermeier, Gollier and Parker (2007), Mitton and Vorkink (2007) and Barberis and Huang (2008)). Donelson and Resutek (2015) find that their earnings uncertainty measure is able to predict future returns for periods exceeding 12 months. They also find that their measure of earnings uncertainty is significantly related to equity analysts' and investors' overly optimistic expectations of future earnings. Thus, establishing a methodology to derive forecasts of earnings uncertainty or risk through the higher moments of earnings in addition to a forecast of the first moment appears to be reasonably useful in various economic settings. Despite the clear motivation to gain information on the higher moments of future earnings, it is a comparably novel area of research (Monahan (2018)).

The terms earnings uncertainty and earnings volatility are often used interchangeably. However, earnings uncertainty refers to the entirety of all moments of future earnings, whereas earnings volatility, i.e. the second moment of future earnings, is only one type of uncertainty. This study focuses on forecasting the second moment of future earnings, i.e. future earnings volatility or variance, although, in theory, the presented approaches are all suited to be applied to even higher moments of future earnings.[1]

As firm-level earnings' variance is not directly observable and neither can be calculated in one point as there is only one realization of firm-level earnings published every year, a proxy for this measure has to be established. This study employs accounting data instead of market data to approximate the earnings variance measure, due to its wider availability thus a wider application of the earnings variance forecasting approaches. Further, Beaver, Kettler and Scholes (1970) find that accounting-based measures of risk are reflected in market-based measures of risk and claim that accounting-based risk measures are better suited to derive forecasts of market-based risk measures. Baginski and Wahlen (2003) establish different accounting-based risk metrics and show that capital markets price the systematic risk in residual income. Also, the two studies by KP and CH exploit accounting panel data by applying quantile regressions to derive out-of-sample earnings quantile forecasts and approximate different moments based on these accounting-based quantiles forecasts.

Higher moments of financial variables such as stock returns are predominantly forecasted via time-series approaches, i.e. ARCH/GARCH-models. For example, Baginski and Wahlen (2003) estimate an abnormal return-on-equity beta, e.g. the systematic risk in residual income from a firm's time-series of residual return-on-equity. Sheng and Thevenot (2012) also exploit the time-series of earnings data and apply GARCH-class time-series volatility models in order to forecast earnings volatility. In a literature review regarding the general forecasting of volatility in financial markets, Poon and Granger (2003) provide vast evidence that time-series forecasting methods based on historical volatility measures perform similarly well as more sophisticated models from the GARCH class or stochastic volatility forecast models. However, as KP point out, such models only capture information in accounting figures to the extent that it is reflected in a firm's individual earnings history. This is not optimal due to three reasons: first, as Donelson and Resutek (2015) state, only if earnings are stable, past earnings volatility will proxy for future earnings uncertainty. Second, cross-sectional variation is not exploited at all (e.g.,

[1] In the remainder of this paper the terms second moment of earnings and earnings variance are used interchangeably.

KP). Third, time-series analyses suffer from higher data requirements (e.g., Bradshaw, Drake, Myers and Myers (2012)). The two studies by KP and CH tackle these issues and forecast higher moments of future earnings by applying cross-sectional quantile regression approaches. In line with these two studies, this study as well implements a cross-sectional instead of a time-series approach.

As there are two existing approaches which use accounting data and derive forecasts for higher moments of future earnings via a cross-sectional forecasting approach, i.e., the two approaches by KP and CH, those are the two approaches the residuals-based earnings variance proxy, which will be presented in the following, will be benchmarked against. KP use the difference between the forecasted 75^{th} and the 25^{th} percentile to construct the interquartile range (IQR) as one of their earnings uncertainty measures and claim this measure to be proportional to the variance of future earnings. CH follow a similar approach as KP. However, they differ from KP in that they (a) construct their measures of higher moments differently and (b) include different predictor variables.[2] In contrast to KP, they model the return on equity and not firm-level earnings. Similar, to KP they implement a quantile regression approach, although they model 150 different quantiles between 0 and 1, which, in theory, helps covering the possibility of extreme outcomes. Based on the resulting 150 forecasted quantiles, they calculate their different measures of earnings uncertainty.[3]

This study aims to contribute in two ways. First, a residuals-based approach for forecasting future earnings variance is introduced. This approach can serve as an alternative for the quantile-based approaches by KP and CH. Similar to CH and KP this approach employs an accounting-based earnings variance measure in a cross-sectional forecasting approach. The motivation underlying this approach stems from KP who mention that "*Alternatively, one could capture conditional variance (dispersion) in future earnings by regressing the squared (or absolute) value of the residuals from an earnings forecasting model on predictor variables.*".[4] That is, first, the conditional mean of earnings will be modeled and the resulting squared residuals will be interpreted as the variance of the respective observation. Second, these

[2]A detailed explanation of the two approaches by KP and CH follows in the methodological part in section 2.1.2.

[3]A detailed explanation of the construction of the future second moment of earnings follows in section 2.1.2.

[4]Although they report that such forecasts exhibit a high correlation with their quantile-based forecasts, they do not report specific empirical results. Further they do not compare the different approaches which will be done in this study. A detailed explanation of the methodology for the residuals-based forecasting approach follows in section 2.1.2.

squared residuals will be regressed onto different predictor variables.[5] Afterwards, out-of-sample firm-level earnings variance point-forecasts can be derived.

Interpreting the squared residuals from modeling the first moment of an economic variable as a proxy for the respective observation's variance has already been done in other areas of finance research. According to Granger and Ding (1995), such a residuals-based variance approximation has great intuitive appeal due to its simplicity. The approach is based on the idea of ARIMA-(G)ARCH models, where the mean as well as the variance of the process are modeled. For example, Granger and Ding (1995) employ this proxy in the context of analyzing asset returns, while Asseery and Peel (1991) use the proxy in the context of analyzing exchange rate volatility and exports. In this study, such proxy is applied to the earnings measure.

In order to construct this residuals-based variance proxy, a model for the first moment of future earnings, from which the residuals will be retrieved, is needed. Thus, other ways than earnings forecast models to gain information about future earnings such as analyst forecasts or machine learning approaches (e.g., Cao and You (2020) and Hendriock (2022)) are not applicable for this approach. Most of the earnings forecast models employ a cross-sectional approach and thus, by design, are superior to analyst-based earnings forecasts in terms of coverage (e.g., Hou, Van Dijk and Zhang (2012) and Li and Mohanram (2014)). A popular earnings forecast model is the one developed by Hou, Van Dijk and Zhang (2012), hereafter HVZ. While their model beats analyst-based earnings forecasts in terms of coverage, forecast bias and earnings response coefficient, it performs worse with regard to the forecast accuracy. Additionally, Gerakos and Gramacy (2013) note that the HVZ model exhibits forecast errors similar to or even worse than a random walk model, questioning the suitability of the HVZ model. In response Li and Mohanram (2014) propose two new earnings forecast models, namely the earnings persistence (EP) model and the residual income (RI) model. They provide evidence that both their models outperform the original HVZ model in terms of forecast bias, accuracy and earnings response coefficient. Of these mechanical earnings forecast models the RI model is typically found to perform best in terms of forecast accuracy (e.g., Li and Mohanram (2014) and Hendriock (2022)). Evans, Njoroge and Yong (2017) as well as Tian, Yim and Newton (2021) provide evidence that using the least absolute deviation method, i.e., median regressions, further improves the earnings forecast

[5] Although KP and CH use the same set of predictor variables for forecasting different moments of future earnings, in theory, the quantile-based approach as well as the residuals-based approach are able to employ different predictor variable sets depending on the moment that is forecasted.

performance. However, the following framework draws on using the residuals from modeling the first moment of earnings, i.e., mean and not median earnings, in order to approximate the second moment of earnings. Thus, relying on median regression is not applicable in this study. Based on the presented findings from former studies, in this study the forecast model for the first moment of future earnings will be the RI model by Li and Mohanram (2014).[6]

Next to applying the residuals-based variance proxy to earnings, as a second contribution this study tests new predictor variable sets in order to further enhance the forecast accuracy in addition to the predictor variables already used in the two studies by KP and CH. Thus, the empirical analysis in this study includes three different variance proxies, i.e. the two quantile-based proxies by CH and KP as well as the residuals-based proxy. Further, five different predictor variable sets will be included in the analysis, i.e. the two sets from the studies by CH and KP as well as three new sets. Combining each variance proxy with each predictor set results in 15 different variance forecasts which will then be evaluated and compared with each other. Thus, the main objective of this study is to compare the forecasting performance of quantile-based earnings variance proxies, e.g. the proxies by CH and KP, with the presented residuals-based earnings variance proxy.

In general, assessing the accuracy of firm-level earnings variance point-forecasts is difficult. Whereas a mean earnings forecast can easily be evaluated against the realized earnings figure of the forecasted period, such approach is not applicable for forecasts of higher moments of earnings such as the future earnings variance. That is, for each firm-year only one earnings figure can be observed. As the calculation of the realized variance requires at least two realizations, a direct comparison between forecasted and actual values is not feasible. Thus, in order to evaluate earnings variance forecasts, an approximation of the realized variance has to be established. The first evaluation method in this study, the industry-level evaluation, which was already applied by CH, overcomes this problem by aggregating the firm-level forecasts to industry-level forecasts via the law of total variance described by Brillinger (1969). This forecasted industry variance can then be benchmarked against the realized industry variance using a Mincer and Zarnowitz (1969) regression and the resulting

[6]Note, that the mean earnings forecast model does not only find application for implementing the residuals-based earnings variance forecast approach, but also during the evaluation of the different variance forecasts.

out-of-sample R^2 as a forecast accuracy measure.[7] They compare their quantile-based forecasting approach with three other approaches, e.g., the approach used by KP, an extension of the historical matched samples approach by Donelson and Resutek (2015) and an historical firm-level approach. CH find that their quantile-based predictions perform better than all of the three alternative measures in such an industry-level evaluation. They analogously apply the aforementioned procedure to both skewness and kurtosis and derive similar results. As this procedure applies a feasible approximation of the non-observable realized standard deviation, it will be included as an evaluation approach in this study as well. In line with the study by CH, the results of this study indicate that the variance proxy used in their study outperforms the other two approaches in terms of forecast accuracy. Nevertheless, the results of that evaluation approach have to be treated with caution. As CH mention, their quantile-based approach is able to capture the likelihood of extreme outcomes, because they derive their moments of future earnings through a series of 150 forecasted earnings quantiles. However, if only the second moment of future earnings is investigated the incorporation of the extreme quantiles is questionable as they might not be as relevant for the approximation of the earnings' variance as for even higher moments. Thus, the forecasts of these extreme quantiles potentially spoil the approximation of the second moment. The empirical results support this hypothesis as the summary statistics for the variance forecasts reveal that the approach by CH is prone to produce extremely large firm-level variance forecasts. Subsequently, it seems that through the aggregation of firm-level forecasts to industry-level forecasts the influence of these extreme forecast values is reduced so that the poor performance on firm-level is masked. The outperformance of the quantile-based variance forecast approach by CH thus might only stem from the aggregation of the variance forecasts on industry-level, not from the forecasting approach itself thus questioning the suitability of this industry-level evaluation approach.

In order to further investigate that hypothesis, in this study a second evaluation method is implemented that tackles this problem and approximates the realized variance on firm-level. To do so, the realized variance on firm-level will be approximated in line with the idea behind the residuals-based variance proxy as the squared difference between the forecasted and the realized firm-level earnings measure. Then the firm-level earnings variance forecasts will be evaluated against

[7] Note that this industry-level evaluation method jointly evaluates mean and variance forecasts. Thus, it is especially important that the mean forecasts are the same for all approaches in order to isolate the performance of the variance forecasts.

that approximated realized variance. Again, the out-of-sample R^2 from a Mincer and Zarnowitz (1969) regression of the realized variance on the forecasted variance is used as a forecast accuracy measure. A similar firm-level evaluation was already implemented by Donelson and Resutek (2015). When using the out-of-sample R^2 from Mincer and Zarnowitz (1969) regressions, the results of this study imply that the residuals-based variance proxy is suited best in terms of forecast accuracy and thus outperforms the quantile regression-based approaches. Further, the results of this study indicate that this outperformance of the residuals-based variance proxy is driven by its robustness to producing extreme variance forecasts which is more likely to happen when using a quantile-based earnings variance proxy.

Regardless of the underlying earnings variance proxy, the results indicate that including the predictor variable book-value of equity further enhances the variance forecast accuracy. That is, extending the sets of predictor variables from the studies by KP and CH by the variable book-value of equity results in more accurate earnings variance forecasts. Mixed results regarding the forecasting performance when including industry-fixed effects in the variance forecast models question the necessity to incorporate such industry dummies in the earnings variance forecast models as well as the importance of industry membership for the second moment of future earnings.

Finally, an approach to test the economic relevance of the earnings variance forecasts, similar to the studies by KP and CH is implemented. That is, it is tested whether the information captured by the earnings variance forecasts are relevant to equity prices or bond ratings. For example, KP assess the relation of the predicted IQR, skewness and kurtosis with equity and debt market measures and find that their forecasts of higher moments are related to equity and credit risk ratings, future return volatility, credit spreads and analyst based measures of earnings uncertainty even after including control variables and conclude that their forecasts possess incremental information. Similar to KP, CH also provide evidence for the economic relevance of their quantiled-based predictions of the higher moments of future return on equity. They regress a number of equity-market and credit-market variables on their predictions and control variables and find that their predictions are related to both the equity- and the debt-market. More specifically, they provide evidence that equity prices are increasing (decreasing) in the standard deviation and skewness (kurtosis) of future return on equity and credit spreads are increasing (decreasing) in the standard deviation and kurtosis (skewness) of lead return on assets. These findings are in line with the model by Pástor and Veronesi (2003) who predict such relationship and contradict the model by Merton (1987) who predicts a negative relationship between stock return volatility and equity prices. In conclusion, their study indicates that accounting-based data contains information about future earnings uncertainty and that this uncertainty is priced. Similar, the results of this study show that information about future earnings variance derived from accounting data is priced in equity markets, whereas it seems that rating agencies do not make use of these information when rating bonds. CH hypothesize that this is caused by rating agencies being more concerned with past earnings variance and less with future earnings variance.

The remainder of this paper is structured as follows: Section 2 presents the methodology applied in this study. Section 3 describes the data, section 4 shows the results from the empirical analysis and discusses these. Section 5 concludes.

2 Methodology

The analysis is divided into two parts. First, section 2.1 introduces the forecast models for the conditional first (section 2.1.1) and second (section 2.1.2) moment of future earnings. That is, in section 2.1.1, the conditional mean earnings forecast model by Li and Mohanram (2014) is presented. Section 2.1.2 then presents the two existing methods to derive forecasts for the conditional second moment of future earnings by KP and CH and subsequently presents the methodology for the residuals-based earnings variance proxy. Section 2.2 presents the different evaluation techniques to compare the resulting forecasts of the future earnings variance. Two evaluation methods that aim to assess the forecast accuracy will be discussed. Both methods apply the out-of-sample R^2 as an evaluation metric, first using an industry-level test and second using a firm-level test. Finally, a methodology to assess the economic relevance of these forecasts will be presented in section 2.3. That is, it will be examined whether the forecasted information are captured in equity prices or bond ratings.

2.1 Conditional Mean Earnings and Earnings Variance Forecasts

2.1.1 Forecasting The First Moment of Future Earnings

Throughout the following empirical analysis, information about the first moment of future earnings, i.e. future mean earnings, is needed. Thus, first a method to derive those is presented. As elaborated before, the best suited earnings forecast model in terms of forecast accuracy is the RI model by Li and Mohanram (2014), which can be expressed by the following estimation equation:

$$\begin{aligned} Earn_{i,t+\tau} = \beta_0 + \beta_1 Earn_{i,t} + \beta_2 d^-_{i,t} + \beta_3 d^- Earn_{i,t} \\ + \beta_4 BkEq_{i,t} + \beta_5 TACC_{i,t} + \epsilon_{i,t+\tau}, \end{aligned} \tag{1}$$

where $Earn$ reflects earnings, d^- is an indicator variable equal to one if $Earn_{i,t} < 0$ and zero otherwise, $d^- Earn$ is an interaction term of the dummy variable d^- and $Earn$, $BkEq$ is the book-value of equity, $TACC$ reflects total accruals, t represents the time index, τ is a time constant and ϵ is the error term. In line with Li and Mohanram (2014), earnings are defined as earnings excluding special items.

If not stated otherwise, throughout the entire study per-share measures are applied, that is, all variables are scaled by the number of common shares outstanding. In line with former research on mean earnings forecast models a cross-sectional rolling OLS regression approach with a window length of ten years in order to train the model is implemented. More specifically, for each window the annual data from year $t-9$ to year t is used to estimate the model's parameter estimates. To derive forecasts for the first moment of future earnings one year ahead, i.e. $\tau = 1$, the retrieved parameter estimates are multiplied with the realized data from year t in order to obtain firm-specific mean earnings estimates for year $t + \tau$.

Another option for deriving mean earnings forecasts is implementing the OLS regression approach by Li and Mohanram (2014) not with their predictor variables, but in combination with the five predictor variable sets used for forecasting the second moment of future earnings. Thus, the forecasts for the first and second moment will be based on the same predictor variables. However, this leads to problems in the evaluation. If the mean earnings forecasts throughout the evaluation differ in addition to the different variance forecasts, the presented evaluation methods do not independently assess the performance of the variance forecasts, but jointly evaluates the forecasts for the first and second moment. Thus, in order to secure a isolated evaluation of the earnings variance forecasts, mean earnings forecasts are firm-year forecasts retrieved from the RI model by Li and Mohanram (2014).

2.1.2 Forecasting The Second Moment of Future Earnings

This section describes the two existing methods to derive forecasts for the conditional second moment of future earnings by KP and CH as well as the methodology for the approach based on squared residuals proxy. The three different approaches to derive an earnings variance forecast will be referred to as *earnings variance proxies*. More precisely, the method to construct such proxy by KP will be referred to as the *KP Proxy*, the method by CH as the *CH Proxy* and the method applying squared residuals as the *SR Proxy*. The three different methods will then be combined with five different predictor variable sets, so that finally, a pool of 15 earnings variance forecast models is established and 15 different earnings variance forecasts can be derived and evaluated.

The Earnings Variance Proxy After KP

In order to construct their earnings variance proxy, KP make use of the quantile regression technique (Koenker and Bassett Jr. (1978)). Equivalently in this study, the following estimation equation for both the 25^{th} and the 75^{th} percentile is applied, i.e. for $q = 0.25$ and $q = 0.75$:

$$\begin{aligned} Q_q(Earn_{i,t+\tau}|\cdot) = {} & \beta_0^q d_{i,t}^+ + \beta_1^q d_{i,t}^- + \beta_2^q d_{i,t}^+ TACC_{i,t} + \beta_3^q d_{i,t}^- TACC_{i,t} \\ & + \beta_4^q d_{i,t}^+ OCF_{i,t} + \beta_5^q d_{i,t}^- OCF_{i,t} + \beta_6^q d_{i,t}^+ SPI_{i,t} + \beta_7^q d_{i,t}^- SPI_{i,t} \\ & + \epsilon_{i,t+\tau}. \end{aligned} \tag{2}$$

where OCF is operating cash flow, SPI is special items and d^+ is an indicator variable equal to one if $Earn_{i,t} \geq 0$ and zero otherwise. Additionally, the model includes industry fixed effects. The interaction of all variables with a loss as well as a non-loss dummy enables a differentiation between loss and non-loss firms with regard to the relationship between the predictor variables and the quantiles modelled. Further they include industry dummies, which, if relevant for the model, represent different earnings characteristics depending on the industry a company operates in. Both of the two quantiles will be modelled by five different predictor sets. Those five different predictor sets will also be used for the other two forecasting approaches. Predictor set I, i.e. the one displayed in the estimation equation above, and II are the original predictor sets used in the studies by KP and CH, respectively. Since CH do not incorporate industry dummies in their model whereas KP do, predictor set III represents the predictor set by CH extended by industry dummies. Predictor sets IV and V add the variable book-value of equity as a size proxy to respective predictor sets I and III. In order to conserve space only the estimation equation for the first set of predictor variables will be included. However, a detailed overview over the five different predictor variable sets is given in table A9 in the appendix A.2.

The model is re-estimated on a rolling basis with window length of ten years, leading to a series of parameter estimates for the two quantiles. Afterwards, equal to the methodology for deriving forecasts for the first moment of future earnings, the parameter estimates are multiplied with the realized data from year t to derive out-of-sample quantile forecasts for the 25^{th} and the 75^{th} percentile. Then, for each firm-year the difference between the 25^{th} and the 75^{th} percentile, i.e. the IQR, is calculated, which, according to KP, is assumed to be proportional to the variance of the respective observation. However, under the assumption of a normal distribution, in order to transform the IQR into a variance measure, the IQR has to be divided by 1.35 and the resulting measure has to be squared. Doing so produces the firm-level out-of-sample earnings variance forecast, i.e. a forecast for the second moment of future earnings, based on the KP proxy.

The Earnings Variance Proxy After CH

The method proposed by CH to derive the earnings variance forecasts is also based on quantile regressions (Koenker and Bassett Jr. (1978)). First, for a range of quantiles Q, i.e. 150 quantiles between 0.01 and 0.99 with equal increments, the following equation is estimated:

$$\begin{aligned} Q_q(Earn_{i,t+\tau}|\cdot) = {} & \beta_0^q d_{i,t}^+ + \beta_1^q d_{i,t}^- + \beta_2^q d_{i,t}^+ TACC_{i,t} + \beta_3^q d_{i,t}^- TACC_{i,t} \\ & + \beta_4^q d_{i,t}^+ OCF_{i,t} + \beta_5^q d_{i,t}^- OCF_{i,t} + \beta_6^q d_{i,t}^+ SPI_{i,t} + \beta_7^q d_{i,t}^- SPI_{i,t} \\ & + \epsilon_{i,t+\tau}. \end{aligned} \tag{3}$$

This model is re-estimated on a rolling basis with window length of ten years, leading to a series of 150 out-of-sample quantile forecasts for each firm-year. Again, equally to the methodology for the KP proxy, the estimation equation is combined with each of the five predictor sets for each of the 150 different quantiles. Then the parameter estimates are multiplied with the realized data from year t to derive out-of-sample quantile forecasts for the 150 quantiles resulting in 150 quantile forecasts for year $t+\tau$. Afterwards, in line with CH, the quantile forecasts are rearranged using the approach by Chernozhukov, Fernández-Val and Galichon (2010), so that they do not cross, i.e. that the firm-year quantile forecasts are monotonically increasing with the quantiles. Then, to calculate the firm-level variance forecast, for each series of out-of-sample firm-year quantile forecasts, the squared mean of the firm-year quantile forecasts is subtracted from the mean of the squared quantile forecasts. This is the expected second moment of earnings according to CH, i.e.:

$$VAR(Earn_{i,t+\tau}) = \frac{1}{Q}\Sigma_{q=1}^{Q}(Q_q(Earn_{i,t+1}|\cdot))^2 - (\frac{1}{Q}\Sigma_{q=1}^{Q}(Q_q(Earn_{i,t+1}|\cdot)))^2. \quad (4)$$

This procedure then results in five different firm-year earnings variance forecasts, each based on one of the five different predictor sets.

The Residuals-Based Earnings Variance Proxy

For the construction of the the residuals-based earnings variance proxy, first, an earnings forecast model is needed. Section 2.1.1 already introduced the methodology applied to derive firm-year mean earnings forecasts by using the RI model by Li and Mohanram (2014). In a second step, the residuals from modelling the first moment of future earnings are retrieved, annually winsorized at the 1^{st} and 99^{th} percentile, squared and then used as a proxy for the variance of the respective observation, so that:

$$Var(Earn)_{i,t} = (\epsilon_{i,t})^2. \quad (5)$$

To derive a forecast of the second moment of future earnings, the approach follows the suggestion by KP to regress the squared residuals from a mean earnings forecasting model on different sets of predictor variables. In other words, the squared residuals are now themselves modelled. This translates to the following estimation equation in order to model future earnings variance Var with the predictor variable set I:

$$\begin{aligned} Var(Earn)_{i,t+\tau} = {} & \beta_0 d^+_{i,t} + \beta_1 d^-_{i,t} + \beta_2 d^+_{i,t} TACC_{i,t} + \beta_3 d^-_{i,t} TACC_{i,t} \\ & + \beta_4 d^+_{i,t} OCF_{i,t} + \beta_5 d^-_{i,t} OCF_{i,t} + \beta_6 d^+_{i,t} SPI_{i,t} + \beta_7 d^-_{i,t} SPI_{i,t} \\ & + \epsilon_{i,t+\tau}. \end{aligned} \quad (6)$$

From modelling the variance of future earnings, the resulting parameter estimates are retrieved and then multiplied with the realized values from period t in order to obtain firm-specific earnings variance forecasts for year $t + \tau$.

Implementing the former three approaches to derive earnings variance forecasts with the five different predictor variable sets, leads to 15 different firm-year earnings variance forecasts. The following section 2.2 presents the evaluation techniques used to compare the forecasts with each other. It is important to emphasize that the evaluation works in two dimensions. On the one hand, the three different variance proxies evaluated and compared, while on the other hand the suitability of different sets of predictor variables will be analyzed.

2.2 Evaluation of the Earnings Variance Forecast Accuracy

Before presenting the different evaluation methods for the earnings variance forecasts, it is important to note that the evaluation of earnings variance point-forecasts on a firm-year level is not straightforward. That is, it is not possible to observe a realized variance in one point and thus it is not possible to evaluate the forecast in comparison to a realized value as it is, for example, possible when evaluating forecasts of the first moment of earnings for which a realized value as benchmark can actually be observed. Both evaluation methods presented are based on calculating the out-of-sample R^2 from Mincer and Zarnowitz (1969) regressions. The first evaluation is based on an industry-level test as implemented in the study by CH. The second evaluation operates in firm-level. A similar approach is also implemented by Donelson and Resutek (2015) who apply it to evaluate their predictions of earnings uncertainty or French, Schwert and Stambaugh (1987) who use it in the context of return volatility.

2.2.1 Industry-Level Forecast Accuracy Evaluation

CH implement an industry-level approach as it is possible to observe the realized earnings variance of an industry with multiple firms in a given year. They make use of the law of total variance in order to compute the forecasted industry earnings standard deviation in each year, which can then be evaluated against the realized industry earnings standard deviation in the respective year. More specifically, the forecasted industry standard deviation is estimated as the square-root of the sum of the variance of the firm-level earnings forecasts and the industry mean of the forecasted variance, i.e.:

$$\sqrt{VAR(Earn_{IND,t+1}|\cdot)} = \sqrt{VAR(\widehat{Earn}_{i,t+1}|\cdot) + \widehat{VAR}(Earn_{i,t+1}|\cdot)}, \qquad (7)$$

where $\widehat{X}$ represents the forecasted value of variable X. Afterwards, the realized industry standard deviation is regressed on the predicted industry standard deviation.

This approach implements the idea of a Mincer and Zarnowitz (1969) regression on industry-level and the resulting out-of-sample R^2, representing the percentage of the variation in the realized variance captured by the variance forecast, for each of the 15 earnings variance forecasting approaches can then be compared. A higher R^2 represents more accurate industry-level variance forecasts. For the computation of the forecasted industry-level standard deviation according to the law of total variance, mean earnings forecasts are needed which are derived via the RI mean earnings forecast model by Li and Mohanram (2014).

2.2.2 Firm-Level Forecast Accuracy Evaluation

The second evaluation method applies firm-level Mincer and Zarnowitz (1969) regressions to evaluate the variance forecasts by comparing the resulting out-of-sample R^2s. The basic idea behind this approach is to regress a realized value onto the forecasted value for the same period and to interpret the resulting R^2 as a measure of forecast accuracy. As mentioned, it is not possible to observe a realized variance in one point, thus, the realized variance on firm-level has to be approximated in order to enable an implementation of this evaluation approach. The intuition behind that approximation follows the idea of the squared residuals proxy. More specifically, one can approximate the realized variance in one point as the squared difference between the forecasted earnings $\widehat{Earn}_{i,t+\tau}$ and the realized earnings value $Earn_{i,t+\tau}$. This realized variance proxy can then be evaluated against the variance forecast $\widehat{Var(Earn)}_{i,t+\tau}$ for the period $t+\tau$ retrieved from the variance forecast models. In other words, the approximated realized variance is regressed on the predicted variance:

$$(Earn_{i,t+\tau} - \widehat{Earn}_{i,t+\tau})^2 = \beta_0 + \beta_1 \widehat{Var}(Earn)_{t+\tau} + \epsilon_{i,t}. \tag{8}$$

Again, the forecast of mean earnings is based on the RI model by Li and Mohanram (2014), in order to only evaluate the variance forecasts and not jointly the forecasts for the first and the second moment of earnings. As mentioned by Donelson and Resutek (2015), a perfect forecast would result in a slope parameter of 0, i.e. $\beta_0 = 0$ and a parameter estimate of 1, i.e. $\beta_1 = 1$. Thus, larger deviations from these two values indicate poorer forecasts. Additionally, the resulting out-of-sample R^2, will be used as an evaluation metric.

2.3 Assessment of the Economic Relevance

Finally, this study investigates whether the variance forecasts bear economic relevance, that is, whether the variance forecasts are relevant to equity prices or bond ratings.

To assess the economic relevance of the earnings variance forecasts, the evaluation method proposed by CH is applied. A similar analysis was already implemented by KP although the variables investigates were others. The central idea behind the approach is to regress different outcome variable on the variance forecasts and control variables. The chosen outcome variables include the earnings-to-price ration (EP) and the long-term bond-rating (BR_{long}). With this selection equity as well as debt markets are included in the analysis. A statistically significant parameter estimate for the variance forecast implies that the variance forecasts help to explain equity prices or bond ratings. Thus, the estimation equation 9 will be implemented for each variance forecast and both outcome variables. A detailed explanation of the outcome variables as well as the control variables is given in appendix A.1.

$$Outcome_i = \beta_0 + \beta_1 VarianceForecast_i + \sum_n \beta_n Control_{n,i} + \epsilon_i. \tag{9}$$

This equation is re-estimated on a rolling basis with window length of ten years. The outcome and the control variables are winsorized annually at the 1^{st} and 99^{th} percentile.

3 Data

This study's sample consists of the intersection between the databases COMPUSTAT and CRSP and includes annual data of US firms reporting in US Dollar during the period between 1988 and 2021. Financial statements data is retrieved from the COMPUSTAT database while stock price data is taken from the monthly CRSP file. The data preparation follows Li and Mohanram (2014). That is, a reporting lag of 3 months is implemented and each model is estimated at the end of June of the respective year. This requirement causes that financial information of firms with a fiscal year end between April and June in year $t-1$ are not available at the end of June, and thus for the estimation in year t data from April of year $t-1$ until March of year t is used. Further, all variables used in the forecasting models are scaled by the common shares outstanding, if not stated otherwise. A detailed explanation of all variables follows in appendix A.1. All observations with missing entries for any of the variables used in any of the forecasting models are excluded from the sample. Additionally, observations that correspond to a stock price that is smaller than one US dollar and/or zero common shares outstanding are excluded from the sample. Then, following previous literature (e.g., Dechow, Hutton, Kim and Sloan (2012)), financial firms (SIC codes 6,000 to 6,999) are excluded from the analysis as financial statements of these firms are subject to different regulatory frameworks. In order to mitigate the effect of outliers, all variables are winsorized annually at the 1^{st} and 99^{th} percentile. The earnings definition used in this study corresponds to the "core earnings" definition by Li and Mohanram (2014) who define earnings as earnings per share excluding special items. Industries are assigned according to the Fama-French 12-Industries classification (FF12) based on the four-digit SIC code.

Summary statistics for the resulting sample are displayed in table 1.

Table 1: Sample Summary Statistics

	N	Mean	Std	Min	P25	P50	P75	Max
$Earn$	111,746	0.89	2.15	-12.92	-0.13	0.56	1.62	29.32
d^+	111,746	0.70	0.46	0.00	0.00	1.00	1.00	1.00
d^-	111,746	0.30	0.46	0.00	0.00	0.00	1.00	1.00
$BkEq$	111,732	9.18	10.46	-12.53	2.40	6.19	12.46	100.83
$TACC$	111,746	-1.03	2.22	-28.17	-1.49	-0.48	-0.03	10.38
OCF	111,676	1.89	3.15	-6.04	0.02	1.04	2.85	33.67
SPI	111,746	-0.25	0.89	-15.98	-0.16	0.00	0.00	3.35
LEV	111,731	2.46	4.18	-35.72	1.37	1.92	2.89	48.40
$PAYOUT$	111,746	0.28	0.62	0.00	0.00	0.00	0.24	5.69
$PAYER$	111,746	0.33	0.47	0.00	0.00	0.00	1.00	1.00

Table 1 contains descriptive statistics for the entire sample, i.e. the pooled cross-section of firms from 1988 to 2021. It displays summary statistics for all variables of the forecast models for the conditional first and second moment of future earnings. However, in order to conserve space, all interacted variables as well as the industry dummies are omitted in this table.

The sample contains $111,746$ observations. Similar to former studies, the sample includes around 30% of firms with negative earnings (e.g., Hou, Van Dijk and Zhang (2012) and Hess, Meuter and Kaul (2019)). Mean earnings of 0.89 are reported in addition to median earnings of 0.56, indicating a negatively sweked earnings distribution. Around 33% of the observations report dividend payments, which is less than the sample by CH who report dividend payments for 43.8% of the observations. Apart from that, the sample exhibits similar characteristics to former research with regard to average negative accruals and average positive operating cash flows (e.g., Sloan (1996), Barth, Cram and Nelson (2001) and KP). As KP point out, this is due to the fact that accruals include depreciation and amortization and operating cash flows exclude cash flows from investing.

Table A10 in the appendix A.3.1 presents correlations between the forecasting model variables. The results show that the correlations are similar to former studies (e.g., KP and CH). A strong positive relationship between earnings and operating cash flow, a weak positive relationship between earnings and the leverage ratio, a strong negative relationship between earnings and the negative earnings dummy and positive correlations between earnings and dividends paid as well as the payout ratio are reported. However, whereas KP report a positive correlation between earnings and accruals as well as special items of 0.30 and 0.33 respectively, CH only report a correlation between earnings and accruals of 0.22 and this study even finds a slightly negative correlation between the two variables of -0.08. Additionally, this study finds a negative correlation between earnings and special items of -0.1.

Table 2 on the next page presents the number of firms and the number of observations per industry based on the FF12 industry classification. Sufficient observations are crucial for implementing industry-fixed effects as well as one of the evaluation methods regarding the forecasting accuracy, i.e. the industry-level evaluation by CH.

Table 2 shows that a sufficient number of observations in each industry is available to perform all analyses described.

4 Empirical Results

This section presents the empirical results. First the regression results for the forecasting models for the first and second moment of future earnings will be presented in section 4.1. Afterwards, descriptive statistics for the resulting variance forecasts follow in section 4.2. Section 4.3 contains the empirical results from the two different evaluation methods concerned with the forecast accuracy of the variance forecasts. Finally, section 4.4 presents the empirical evidence regarding the economic relevance of the forecasted variance.

Table 2: Fama-French Industry Classification (FF12)

	Number of Firms	Number of Observations
Mining and Construction	314	2,783
Food	272	2,976
Textiles and Printing/Publishing	520	5,647
Chemicals	287	3,301
Pharmaceuticals	1,295	8,903
Extractive Industries	585	5,131
Durable Manufacturers	2,656	27,326
Computers	2,307	18,039
Transportation	777	6,766
Utilities	434	5,463
Retail	1,415	13,407
Services	1,488	12,004

Table 2 contains information about the number of firms as well as about the number of firm-year observation in each industry according to the Fama-French Industry Classification (FF12).

4.1 Regression Results

In the following section, the regression results for the mean earnings forecast model as well as for the earnings variance forecast models will be presented.

4.1.1 Modelling the First Moment of Future Earnings

Table 3 presents the parameter estimates and the respective p-values from the rolling OLS regression using the RI model by Li and Mohanram (2014): As ex-

Table 3: Parameter Estimates for RI model

	$Intercept$	$Earn$	d^-	d^-Earn	$TACC$	$BkEq$
Par. Est.	**0.06***	**0.77*****	**-0.29*****	**-0.23*****	**-0.08*****	**0.01*****
	(0.0512)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0016)

Table 3 contains information regarding the time-series averages of the parameter estimates and the Newey and West (1987) p-values assuming a ten-year lag length from modelling the conditional first moment of future earnings, i.e. mean earnings by using the RI model by Li and Mohanram (2014). To obtain the parameter estimates, a rolling OLS regression approach with a window length of ten years in line with Li and Mohanram (2014) is implemented. ***, **, and * indicate significance at an alpha level of 1%, 5%, 10%, respectively.

pected, all parameter estimates are statistically significant at the 1%-level, except the intercept which is only significant at the 10%-level. Whereas all in all the results of this study are very similar to the one by Li and Mohanram (2014), there are small differences. This study reports a paramter estimate of 0.77 for the earnings variable, whereas Li and Mohanram (2014) report a value of 0.86 implying that this study finds earnings to be less persistent. Additionally, this study finds a larger (smaller) negative influence of the loss-firm dummy (the interaction term between earnings and the loss-firm dummy) on future earnings. Li and Mohanram (2014) report a slightly positive relationship between the book-value of equity and future earnings with a parameter estimate of 0.02 and an even smaller negative parameter estimate for the total accruals of -0.002. This study finds slightly strongr relationships with values of 0.01 and -0.08, respectively.

That the sign of the parameter estimate of the total accruals variable in this study and the study by Li and Mohanram (2014) is negative in both cases is slightly surprising, because of different signs of the correlation between total accruals and earnings in the correlation results. However, only a weak correlation was reported in both studies and overall, the results from the rolling regression for the mean earnings forecast model can be seen as reliable and in line with former research.[8]

4.1.2 Modelling the Second Moment of Future Earnings

In this section, the parameter estimates resulting from modelling the second moment of future earnings will be analyzed. Section A.3.2 in the appendix presents the average parameter estimates over all windows from a rolling regression approach for the three different variance proxy approaches combined with the five different predictor variable sets. All predictor sets except predictor set II contain industry dummies although they are not reported in order to conserve space. The parameter estimates for the 25^{th} and 75^{th} quantile from the forecasting approach by KP are represented by $KP25$ and $KP75$, respectively. Further, the function of the parameter estimates for the 150 quantiles from the forecasting approach by CH are represented by CH in the graphs. Lastly, the parameter estimates from the squared residuals approach are marked as OLS in the graphs.

In almost all cases, except for the loss dummy in predictor set II, III and V, the sign of the parameter estimate from the squared residuals approach, the sign

[8]Untabulated results additionally show that the resulting mean forecasts exhibit very similar forecast errors.

of the slope of the function of the parameter estimates from the approach by CH and the difference between the parameter estimate of the 75^{th} quantile and the 25^{th} quantile from the approach by KP coincide. For example, predictor set I indicates a positive relationship between accruals and future earnings variance of non-loss firms when modelling the future earnings variance via the squared residuals approach. Likewise, the sign of the slope of the function of the parameter estimates for the different quantiles from the approach by CH for accruals of non-loss firms is positive. Finally, also the difference between the parameter estimate of the 75^{th} quantile and the 25^{th} quantile from the approach by KP is positive. In other words, the increasing parameter estimates for the two different quantile approaches of the accruals variable for non-loss firms is consistent with the conditional variance of future earnings of non-loss firms based on the squared residuals approach being positively related to accruals. To facilitate the interpretation of the parameter estimates, it is reasonable to focus on the sign of the parameter estimates from the squared residuals approach since it coincides with the direction of the effect for almost all variables of the other two approaches. Thus, the parameter estimates from modelling the second moment of future earnings directly via the squared residual proxy will be investigated. As mentioned, the respective parameter estimates are reflected by OLS in the graphs.

Predictor sets I and IV, i.e. the predictor sets based on the study by KP, account for differences between loss and non-loss firms. These two sets enable statements about different relationships between predictor variables and future earnings variance for loss and non-loss firms independently. According to the parameter estimates for predictor sets I and IV, higher accruals as well as cash flows are associated with higher (lower) future earnings variance for non-loss (loss) firms. However, these two variables are the only ones exhibiting different signs for the relationship with future earnings variance depending on whether a firm reports a loss or not. Further, the results of all predictor sets indicate the following clear pattern: First, the positive sign for the loss as well as the non-loss dummy reveal that both types of firms tend to have a slightly positive baseline variance of future earnings. Earnings, the book-value of equity and the leverage have a positive influence on future earnings variance regardless whether the firm reports a loss or not, whereas special items, dividends paid and the dividend dummy are associated with a negative influence on future earnings variance.

4.2 Descriptive Statistics for the Variance Forecasts

Summary Statistics for the Earnings Variance Forecasts

Table 4 presents the summary statistics for the different variance forecasts. SR, KP and CH represent the respective earnings variance proxy, whereas the numbers from I to V indicate the set of predictor variables used.

Table 4: Summary Statistics for the Variance Forecasts

	N	Mean	Std	Min	P25	P50	P75	Max
SR(I)	77,713	1.42	2.51	-18.67	0.39	0.73	1.48	66.21
SR(II)	77,713	1.47	2.64	-11.64	0.40	0.78	1.58	66.07
SR(III)	77,713	1.45	2.67	-11.24	0.35	0.74	1.57	65.02
SR(IV)	77,713	1.44	2.63	-14.69	0.33	0.74	1.57	81.65
SR(V)	77,713	1.46	2.67	-9.99	0.32	0.76	1.63	62.81
KP(I)	77,713	0.92	3.44	0.00	0.13	0.28	0.69	214.91
KP(II)	77,713	0.87	2.76	0.00	0.12	0.27	0.69	83.02
KP(III)	77,713	0.88	2.70	0.00	0.12	0.28	0.69	79.34
KP(IV)	77,713	0.98	3.56	0.00	0.13	0.30	0.77	284.54
KP(V)	77,713	0.90	2.60	0.00	0.13	0.30	0.75	71.50
CH(I)	77,713	5.20	118.82	0.01	0.26	0.45	1.04	15,808.27
CH(II)	77,713	4.99	87.40	0.00	0.24	0.42	0.99	5,023.57
CH(III)	77,713	5.06	89.44	0.00	0.23	0.42	1.00	5,307.26
CH(IV)	77,713	5.08	110.83	0.00	0.21	0.44	1.11	14,078.94
CH(V)	77,713	5.02	87.27	0.00	0.21	0.43	1.06	5,224.06

Table 4 contains descriptive statistics for the different variance forecasts, i.e. the pooled cross-section of variance forecasts.

For each forecasting approach, i.e. each combination of variance proxy and predictor variables set, $77,713$ forecasts were derived. Comparing the mean values of the different variance forecasts shows a comparably large average for the variance forecasts based on the proxy by CH ranging from 4.99 up to 5.20. While the variance forecasts based on the squared residuals proxy exhibit significantly lower averages ranging from 1.42 up to 1.47, the forecasts based on the proxy by KP exhibit the smallest average ranging from 0.87 up to 0.98. This high mean values for the forecasts based on the proxy by CH appear to be driven by extremely high values in the extreme percentiles, which becomes evident when comparing the maximal values of the three different variance proxies in comparison to the values of the

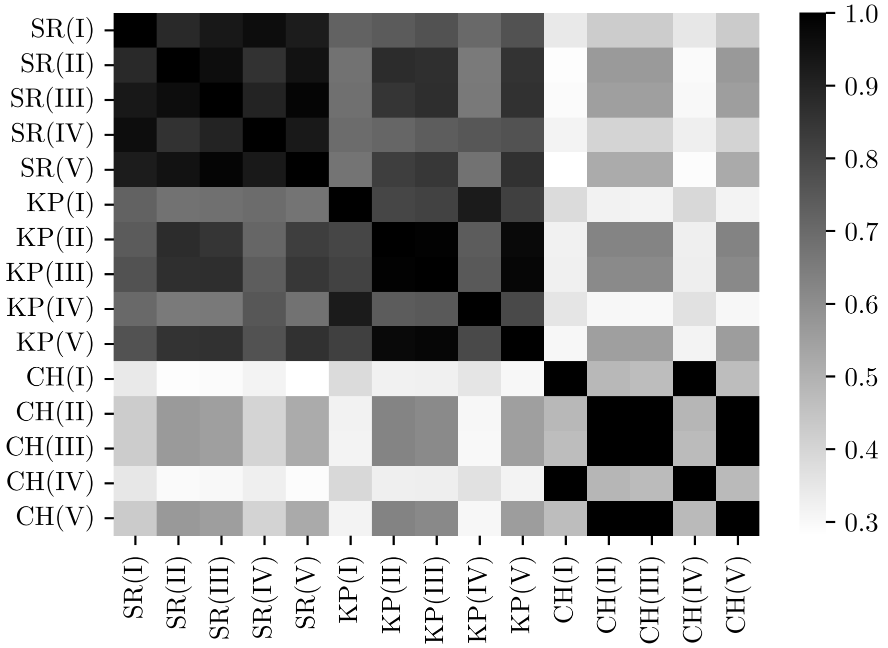
75^{th} percentile. Simultaneously, this results in a higher standard deviation for the respective forecasts. Surprisingly, the two most extreme forecast values regardless of the underlying variance proxy, are the ones derived with the two predictor sets based on the study by KP, i.e. predictor sets I and IV. Thus, the first finding is that the variance proxy by CH is prone to produce extremely high forecasts in the upper percentiles.[9] The general distribution characteristics of the variance forecasts, regardless of the predictor variables used, exhibits the following properties: First, on average the variance forecasts based on the proxy by CH exhibit the highest average values driven by extremely high values in the upper percentiles which, subsequently, also results in comparably high standard deviations. Comparing the minimum, the 25^{th} percentile, the median and the 75^{th} percentile with the forecasts based on the other two variance proxies it becomes evident that for the majority of the variance forecast distributions all three variance proxies exhibit similar characteristics. The forecasts based on the squared residuals proxy come with a slightly lower standard deviation compared with the forecasts based on the proxy by KP and additionally exhibit a lower minimum and smaller maximal values, whereas the 25^{th} percentile, the median and the 75^{th} percentile values appear to be larger. In conclusion, it seems that the forecasts based on the variance proxy by CH exhibit similar characteristics for a large part of the forecast distribution with the forecasts based on the other two proxies, but come with extremely large values in the extreme percentiles. Comparing the forecasts based on squared residuals and the proxy by KP indicates that the latter also comes with comparably high values in the upper percentiles resulting in a larger standard deviation. Nevertheless, values for the 25^{th} percentile, the median and the 75^{th} percentile are lower leading to a smaller average forecasted variance. Additionally, the forecasts based on the squared residuals proxy are the only ones with negative values as can be seen by the negative minimal values. Thus, the forecasts based on the squared residuals proxy have the most extreme minimal values and the least extreme maximal values, but are larger for the 25^{th} percentile, the median and the 75^{th} percentile in comparison to the forecasts based on the other two proxies.

[9]Untabulated results indicate that these high maximal values remain even after yearly winsorizing the variance forecasts at the 1^{st} and 99^{th} percentile.

Cross-Correlations for the Variance Forecasts

The heatmap below indicates the correlations between the different variance forecasts. Again, SR, KP and CH represent the respective earnings variance proxy, whereas the numbers from I to V indicate the set of predictor variables used.

Heatmap Representing the Conditional Variance Forecast Correlations



Analyzing the correlation heatmap confirms some of the former findings from the summary statistics. First, as expected, the forecasts based on the squared residuals proxy as well as the forecasts based on the proxy by KP exhibit a stronger correlation with each other than with the forecasts based on the variance proxy by CH, which is most likely driven by the extremely high values of the forecasts in the upper percentiles for the latter variance proxy. Combining the findings about the correlations with the summary statistics, it appears that the two variance proxies based on the squared residuals approach and the approach by KP result in relatively similar variance forecasts, although, as mentioned, the forecasts based on the variance proxy by KP tend to have higher maximal values, but smaller values for the majority of the remaining distribution. Finally, it is interesting to mention that forecasts which rely on predictor sets I and IV, i.e. the predictor sets based on the study by KP, are characterized by weaker correlations with the other forecasts of the same respective variance proxy, which are all based on variants of the predictor set used by CH. This implies that the choice of predictor variables does matter for the derivation of variance forecasts. However, at this point, neither the summary statistics nor the correlations of the variance forecasts give any information about the forecast accuracy, which will then be investigated in the following part.

4.3 Evaluation of the Forecast Accuracy of the Variance Forecasts

Forecast Accuracy Evaluation After CH

The evaluation approach by CH applies an industry-level test for which the realized variance per industry is regressed onto the forecasted variance per industry as described in the methodological part. Table 5 presents the resulting out-of-sample R^2s:

Table 5: Industry-Level Forecast Accuracy Evaluation

	(I)	(II)	(III)	(IV)	(V)
SR Proxy	0.7333	0.7317	0.7335	0.7342	0.7336
KP Proxy	0.7389	0.7381	0.7381	0.7400	0.7370
CH Proxy	0.7383	0.7714	0.7763	0.7456	0.7775

Table 5 contains information about the forecast accuracy of the respective conditional risk forecast resulting from the forecast accuracy valuation approach by Chang, Monahan, Ouazad and Vasvari (2021). That is, the R^2 for each combination of risk proxy (SR, KP and CH) and predictor variables set ((I)-(VI)) resulting from regressing the realized standard deviation per industry on the predicted standard deviation per industry is reported.

Although, the resulting out-of-sample R^2s exhibit quite similar values, there are some important findings. First, all R^2s are quite high with values ranging from 0.7317 up to 0.7775, which indicates that it is, in general, possible to accurately forecast earnings variance on industry level.

There are two dimensions to consider when deriving statements about the forecasting performance, i.e the underlying variance proxy and the set of predictor variables used to derive the respective forecast. The first comparison investigates the different variance proxies. First, it seems that the risk proxy based on squared residuals always performs worse than the two approaches by CH and KP. Second, the results indicate that the variance proxy by KP itself almost always performs worse than the variance proxy by CH. Only when using the predictor variable set from their own study, i.e. predictor set I, the forecasts appear to be more accurate than the ones based on the proxy by CH. Thus, the first finding based on this evaluation approach is that the earnings variance forecasts based on the forecasting approach by CH outperform the two other approaches regardless of the underlying predictor variables chosen. This is in line with the results of their study since they also find an outperformance of their own approach when applying this industry-level forecast accuracy test.

The second dimension in which the variance forecasts need to be compared are the different sets of predictor variables. Comparing the original predictor sets I and II, except for the variance proxy by CH, the predictor set by KP results in more accurate industry-level variance forecasts. However, regardless of the underlying variance proxy, the forecast accuracy of all variance proxies applying the predictor set by CH can be improved when adding industry dummies to the predictor set II, i.e. the R^2 increases from set II to set III for all variance proxies. That implies that industry dummies improve variance forecasts and indicates that future earnings variance characteristics differs between industries. The second finding is that adding the variable book-value of equity to the set of predictor variables increases the forecast accuracy. In detail, comparing predictor sets I and IV, i.e. the predictor set from the study by KP without (I) and with (IV) the variable book-value of equity, an increase in the R^2 can be reported regardless of the underlying variance proxy. Further, comparing predictor sets III and V, i.e. the predictor set from the study by CH with industry dummies and without (III) and with (V) the variable book-value of equity, an increase in the R^2 can be reported for two out of the three variance proxies. Thus, in five of the six cases including the variable book-value of equity improved the forecast accuracy.

Finally, the most accurate forecasts seem to be derived when using the variance proxy by CH and combining it with the predictor set including the variables from their own study and adding industry dummies as well as the book-value per share variable, e.g. predictor set V, resulting in a R^2 of 0.7775. This is in line with the study by CH, how also find their variance proxy to perform best.

Nevertheless, there is one critical characteristic coming with the evaluation approach by CH. As mentioned, it is based on an industry-level test and does not examine the forecasts on firm level although that is arguably the more relevant application of these forecasts. Table 4 gave information about the extremely high values in the upper percentiles for the forecasts by CH. These extreme outliers might be masked by such an industry-level evaluation which then can result in a better performance of a specific forecasting approach in comparison to others on industry-level compared to firm-level. Another disadvantage of this industry-level evaluation method becomes evident when looking at the results in table 5. As mentioned the values of the R^2 all fall in a comparably small range between 0.7317 and 0.7775. Identifying differences and deriving patterns, as shown, is possible, but the results imply that all forecasting approaches result in similarly accurate forecasts. Thus, at this point it remains unanswered whether the variance proxy by CH actually performs better or just benefits from an evaluation at industry-level. In order to investigate that problem further, this study implements another evaluation approach which does not rely on an industry-level test. More specifically, the out-of-sample R^2 from regressing the realized earnings variance onto the forecasted earnings variance, i.e. a Mincer and Zarnowitz (1969) regression on firm-level, will be analyzed in the following section. To do so, this study establishes a proxy for the realized earnings variance based on the idea of the squared residuals approach and interpretes the squared difference between the forecasted mean earnings and the realized mean earnings as the realized earnings variance. The results of this firm-level evaluation, which overcomes the problems of the industry-level approach, are presented in the following.

Forecast Accuracy Evaluation Using Mincer and Zarnowitz (1969) Regressions

This evaluation methods investigates the variance forecasts on a firm-level and not on industry-level as the former method by CH. Table 6 on the next page reports the results from the Mincer and Zarnowitz (1969) regressions as described in section 2.2.2.

Donelson and Resutek (2015), who implement a similar evaluation, mention that the better the forecasts reflect the realized values, the closer the intercept (β_0) will be to 0 and the closer the slope parameter (β_1) will be to 1. Additionally, the resulting out-of-sample R^2 gives information about the accuracy of the forecasts, i.e. how well the variance forecasts capture the variation in the realized variance.

Regarding the negative intercepts β_0 and the comparably large slope coefficients β_1, it seems that the forecasts based on the squared residuals proxy are too low for small future earnings variances and then increase quickly and become too large for higher values of realized future earnings variance. This is in line with the findings from the summary statistics which reflect extremely low and even negative values for the minimum, but then across the majority of the distribution higher values compared to the other two approaches. On the other hand the positive intercepts β_0 and the large slope coefficients β_1 for the variance proxy by KP indicate that the baseline

Table 6: Firm-Level Forecast Accuracy Evaluation

		(I)	(II)	(III)	(IV)	(V)
SR Proxy	β_0	**-1.75**	**-1.75**	**-1.63**	**-1.90**	**-1.73**
		(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
	β_1	**3.13**	**3.05**	**3.04**	**3.16**	**3.06**
		(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
	R^2	0.1266	0.1243	0.1248	0.1429	0.1306
KP Proxy	β_0	**0.97**	**0.30**	**0.28**	**0.58**	**0.03**
		(0.0000)	(0.0000)	(0.0002)	(0.0000)	(0.6700)
	β_1	**1.77**	**2.73**	**2.74**	**2.08**	**2.91**
		(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
	R^2	0.0924	0.1141	0.1119	0.1315	0.1248
CH Proxy	β_0	**2.36**	**2.00**	**2.01**	**2.35**	**1.99**
		(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
	β_1	**0.05**	**0.18**	**0.17**	**0.05**	**0.18**
		(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
	R^2	0.0205	0.0608	0.0596	0.0223	0.0616

Table 6 contains information about the forecast accuracy of the respective conditional risk forecasting approach. That is, the parameter estimates for the intercept and the variance forecast, the respective p-values as well as the R^2 for each combination of risk proxy (SR, KP and CH) and predictor variable sets ((I)-(VI)) resulting from regressing the realized variance for each firm-year observation, which is calculated as the sqaured difference between the realized and the forecasted mean earnings in $t+1$, on the predicted variance per firm-year observation is reported.

variance is on average forecasted too large and then increases too fast with increasing realized future earnings variance values. With regard to the variance proxy by CH, the extremely large intercepts β_0 and the comparably small slope coefficients β_1 indicate a way too large baseline variance which then only increases little with an increasing realized future earnings variance. In conclusion, the squared residuals proxy produces forecasts with a negative baseline variance, but then increase too quickly. The proxy by KP results in a larger than zero baseline variance forecast and then also increases too quickly and the proxy by CH produces a way too high baseline variance and then only varies little with an increasing realized earnings variance. These findings are in line with the summary statistics of the variance forecasts presented in table 4. Whereas these findings give information about the distribution of the respective earnings variance forecasts, the following investigates the resulting forecast accuracy based on the out-of-sample R^2, as the resulting R^2 reflects how much of the variation of the realized future earnings variance is explained by the respective forecast. Again, the first dimension for the comparison are the different variance proxies.

Comparing the R^2s reveals that for all predictor variable sets the proxy based on squared residuals performs better than the one by KP which itself performs better than the one by CH. Relating that finding to the distributional characteristics of the variance forecasts, the following conclusions can be drawn. As seen before, the smaller variance and the smaller range between the minimal and maximal value for the forecasts based on the squared residuals proxy seems to translate to better forecasts compared to the proxy by KP and CH. Further, as the parameter estimates from the Mincer and Zarnowitz (1969) regression indicate, the forecasts of the squared residuals proxy have a much smaller baseline variance which is beneficial, because a general problem with the other two approaches seems to be an overestimation of the future earnings variance.

Comparing the different predictor sets gives the following insights. First, in line with the former evaluation method, the predictor variable book-value of equity improves the variance forecasts regardless of the underlying variance proxy. Second, the evidence on the inclusion of industry dummies vanishes as industry dummies only improve forecasts based on the squared residuals variance proxy, i.e. in one out of three cases. Third, predictor set IV, i.e. the predictor set used in the study by KP extended by the interacted predictor variable book-value of equity, results in the most accurate forecasts when using the squared residuals variance proxy or the proxy of their respective study. But, the results imply further that when using the variance proxy by CH, the best predictor set is set V, which is based on the predictor set used in their study extended by industry dummies as well as the predictor variable book-value of equity.

Conflicting the industry-level evaluation approach by CH, this firm-level evaluation reveals the variance proxy based on the squared residuals approach in combination with predictor set IV to result in the most accurate earnings variance forecasts. Further, regardless of the predictor variables used, the squared residuals proxy is superior to the other two quantile-based proxies in terms of forecasting the conditional second moment of future earnings. However, it needs to be emphasized that this study is solely concerned with forecasting the second moment of future earnings. The two studies by KP and CH also aim to forecast even higher moments. It is in that setting that CH find their variance proxy to outperform the one by KP. As this study finds the latter one to perform better in forecasting the second moment, it is possible that the former approach is able to better capture even higher moments and thus to better forecast the totality of higher moments of future earnings, i.e. future earnings uncertainty. On a theoretical level this hypothesis is backed by the fact that the approach by CH estimates and forecasts 150 quantiles which are then used to construct forecasts for different moments of future earnings. Thus, this approach is able to capture the possibility of extreme outcomes, which are especially relevant for higher moments. In contrast, KP construct their forecasts of different moments of future earnings based on only 11 quantiles. With regard to forecasting only the second moment of future earnings, the smaller amount of forecasted quantiles, i.e. KP only use two quantiles to construct their IQR measure, might be beneficial as the calculation does not get polluted by extreme outliers which are more likely to appear in the extreme quantiles. As mentioned before, the occasionally appearing extreme outliers in quantile forecasts and subsequently the extreme outliers for the variance forecasts from the proxy by CH might be masked by an industry-level evaluation, but become evident in a firm-level evaluation setting as the presented one. Thus, whereas the approach by CH appears to perform worse compared to the approach by KP with regard to forecasting the second moment of future earnings due to the possible influence of the extreme quantiles, exactly this consideration of the tails of the earnings distribution might be beneficial when forecasting even higher moments such as skewness and kurtosis. As the intuition behind the approach of the squared residuals proxy can theoretically also be applied for even higher moments future research might investigate the performance of forecasting even higher moments with approaches by KP, CH and the squared residuals proxy.

In conclusion, there are two new findings. First, in line with the study by CH, this study finds an outperformance of variance forecasts based on their proxy using an industry-level test. However, the evaluation procedure which examines the different variance forecasts on firm-level could not confirm this finding and even contradicts it. That is, the firm-level results indicate that forecasting future earnings variance using the residuals-based earnings variance proxy based on squared residuals from modelling the first moment of earnings leads to the most accurate firm-level forecasts. A possible explanation for that is that the industry-level consolidation masks the influence of extreme variance forecasts, mostly prominent for the quantile-based variance forecasts, which then do not negatively affect the results as much as during a firm-level test. In other words, the residuals-based earnings variance proxy outperforms the other two likely due to its robustness against producing forecast outliers.

Second, this study confirms that forecasts based on the predictor variable sets by KP and CH can produce reliable variance forecasts. However, these forecasts can be further improved by adding the book-value of equity as a predictor variable for the future earnings variance. Results on the inclusion of industry dummies are mixed.

4.4 Evaluation of the Economic Relevance of the Variance Forecasts

After investigating the forecast accuracy of the different variance forecasts, this section deals with the economic relevance of these forecasts, i.e. whether the information are relevant to equity and/or debt markets. To do so, different outcome variables are regressed on the variance forecasts and some control variables. A significant relationship between the outcome variable and the variance forecast implies that the forecast contains information that help explain the respective outcome variable. As mentioned in the former section, the most accurate forecasts are derived when applying the predictor variable set IV for the variance proxy based on the squared residual proxy and the proxy by KP and predictor variable set V for the variance proxy by CH. Thus, forecasts for the three different variance proxies based on the respectively best performing predictor set will be examined in this section. The outcome variables include the equity market variable earnings-to-price ratio as well as the debt market variable the long-term bond rating. As mentioned in the methodology section, this evaluation method is taken from CH. However, already KP performed a similar analysis.

$Outcome\ Variable:\ Earnings - to - Price - Ratio$

The first outcome variable that is investigated is the earnings-to-price ratio. The calculation of this variable as well as all control variables is explained in detail in the appendix A.1.[10] Table 7 on the next page presents the results from a rolling OLS regression with a window length of 10 years.

[10]The variable $HflStd$ is a forecast for the future firm-level standard deviation based on the the historical firm-level standard deviation in order to account for the prominence of time-series approaches. Thus, the test reveals whether the forecasts contain information beyond the ones captured in this time-series measure. The results remain the same if not the standard deviation, but the variance is included.

Table 7: Evaluation of Economic Relevance: $Earings - to - Price - Ratio$

	SR Proxy	KP Proxy	CH Proxy
$Intercept$	**0.12**	**0.11**	**0.11**
	(0.0000)	(0.0000)	(0.0000)
$VarianceForecast_{i,t+1}$	**-0.04**	**-0.03**	**-0.03**
	(0.2006)	(0.0011)	(0.0175)
$HflStd_{i,t+1}$	**-0.02**	**-0.02**	**-0.02**
	(0.0861)	(0.1113)	(0.0968)
$Size_{i,t}$	**0.00**	**0.00**	**0.00**
	(0.1131)	(0.0301)	(0.0773)
$Beta_{i,t}$	**0.01**	**0.01**	**0.01**
	(0.1916)	(0.2068)	(0.2107)
$AnnRet_{i,t}$	**0.04**	**0.04**	**0.04**
	(0.0001)	(0.0001)	(0.0001)
$RetStd_{i,t}$	**-0.63**	**-0.62**	**-0.62**
	(0.0000)	(0.0000)	(0.0000)
R^2	**0.1287**	**0.1340**	**0.1323**

Table 7 contains information about the economic relevance of the respective conditional risk forecast. That is, we report the the time-series averages of the parameter estimates and the Newey and West (1987) p-values assuming a ten-year lag length as well as the R^2 for each combination of risk proxy (SR, KP and CH) and predictor variables set including the interacted variables by Chang, Monahan, Ouazad and Vasvari (2021), industry dummies and the two interacted size variables resulting from regressing the outcome variable EP on the forecasted risk measure and control variables.

First, it appears that future earnings variance is negatively associated with the earnings-to-price ratio, that is, equity prices are increasing in the variance of future earnings. This result is in line with the model by Pástor and Veronesi (2003) and the findings by CH who report the same relationship, although it contradicts the model by Merton (1987). Nevertheless, the relationship between the variance forecast based on the squared residuals proxy and the earnings-to-price ratio is not significant. Further, it becomes evident that the variance forecasts based on the variance proxy by KP has a more significant relationship with the earnings-to-price ratio represented by a parameter estimate which is statistically significant at the 1%-level. Whereas the forecast based on the proxy by CH is only significant at the 5%-level. Note that the resulting R^2 are not comparable to the study by CH, because they also include forecasts of the earnings' mean, skewness and kurtosis in their regressions. Overall, as assumed and in line with CH, the findings suggest that at least the derived variance forecasts for the proxies by KP and CH bear economic relevance and their information is priced.

Outcome Variable: $Long - Term - Bond - Rating$

The debt market outcome variable to be tested is the long-term bond-rating. Table 8 below presents the results from a rolling OLS regression with a window length of 10 years. As table 8 reveals, this study does not find a significant rela-

Table 8: Evaluation of Economic Relevance: $Long - Term - Bond - Rating$

	SR Proxy	KP Proxy	CH Proxy
$VarianceForecast_{i,t+1}$	**-0.05**	**-0.04**	**-0.04**
	(0.4063)	(0.2313)	(0.1598)
$HflStd_{i,t+1}$	**0.30**	**0.30**	**0.35**
	(0.0277)	(0.0455)	(0.0293)
$BP_{i,t}$	**-0.21**	**-0.22**	**-0.23**
	(0.1797)	(0.1691)	(0.1549)
$LNSize_{i,t}$	**-1.22**	**-1.22**	**-1.22**
	(0.0000)	(0.0000)	(0.0000)
$LiabAsset_{i,t}$	**-0.00**	**-0.00**	**-0.00**
	(0.5552)	(0.5399)	(0.5123)
$EbitdaLiab_{i,t}$	**-1.67**	**-1.70**	**-1.56**
	(0.0147)	(0.0120)	(0.0305)
$AnnRet_{i,t}$	**0.52**	**0.51**	**0.52**
	(0.0161)	(0.0160)	(0.0149)
$RetStd_{i,t}$	**12.57**	**12.60**	**12.19**
	(0.0000)	(0.0000)	(0.0000)
R^2	**0.9649**	**0.9649**	**0.9651**

Table 8 contains information about the economic relevance of the respective conditional risk forecast. That is, we report the the time-series averages of the parameter estimates and the Newey and West (1987) p-values assuming a ten-year lag length as well as the R^2 for each combination of risk proxy (SR, KP and CH) and predictor variables set including the interacted variables by Chang, Monahan, Ouazad and Vasvari (2021), industry dummies and the two interacted size variables resulting from regressing the outcome variable BR_long on the forecasted risk measure and control variables.

tionship between any of the variance forecasts and the long-term bond rating, which is in line with CH. Although these weak results appear to be counterintuitive and the reason for that remains unclear, CH hypothesize that investors and rating agencies are possibly more concerned with historical earnings volatility when evaluating credit risk. This hypothesis is backed by the results in table 8. That is, statistically significant parameter estimates for the control variables $HflStd$ and $RetStd$, which both represent historical firm-level standard deviation measures.

In conclusion, the analysis of the economic relevance implies that equity markets price information about future earnings variance and equity prices are increasing in future earnings variance. Further, it seems that debt markets, or more specifically, creators of bond ratings are not concerned with future earnings variance.

5 Conclusion

Information about the future second moment of earnings, i.e. the variance of future earnings, is crucial in various economic settings. This study contributes to the understanding of future earnings variance characteristics in three ways.

First, a residuals-based earnings variance proxy is presented and benchmarked against the two existing cross-sectional, quantile regression-based variance proxies by KP and CH. This variance proxy is based on a suggestion by KP for which the squared residuals from modelling the first moment of future earnings are interpreted as the variance of the respective observation and then modelled again in a second step. The results of firm-level out-of-sample R^2 indicate an outperformance of this approach in comparison to the two quantile regression-based approaches. This outperformance is likely driven by the fact that the approach results in less extreme earnings variance forecasts which becomes not only evident when comparing the standard deviation of the respective forecasts, but also by the less extreme maximal values of the forecasts from this approach in comparison to the quantile regression-based forecasting approaches. It seems that this residuals-based approach is less prone to producing extreme and volatile forecast values which then translates into a better forecast accuracy. Additionally, this study finds that the variance proxy by CH is prone to produce extremely high forecast values, possibly due to the recognition of the extreme quantile forecasts in the calculation of the future earnings variance. Future research might trace back the root of this problem and try to establish a framework that limits the influence of such outliers when forecasting the second moment of future earnings. However, analyzing the performance of the three approaches with respect to forecasting even higher moments such as skewness or kurtosis will be left to future research. In that setting the approach by CH possibly benefits from estimating the extreme quantiles which are more relevant to higher moments such as skewness and kurtosis.

Second, this study emphasizes the difference between an evaluation on industry level in comparison to a firm-level evaluation. As shown, the results differ between the chosen level of aggregation in the evaluation. Since the practical application of information about future earnings variance is mainly concerned with firm-level values, the evaluation method should consider this aggregation as well. However, as mentioned, the possibility to observe realized firm-level earnings variance in one point does not exist, so that CH implemented an industry-level evaluation. This study introduces an approximation for the realized variance on firm-level and benchmarks the forecasts against that proxy using Mincer and Zarnowitz (1969) regressions, similar to the evaluation approach by Donelson and Resutek (2015). In the conceptual approximation of the realized earnings variance on firm-level in one point this study finds a limit, since it is not a perfect approximation, although it seems to be the best one at hand. Future research thus might explore alternative evaluation approaches as well as alternative proxies for the realized earnings variance against which the forecasts can be benchmarked.

Third, this study introduces the possibility of alternative predictors for future earnings variance which were neither covered in the study by KP nor by CH. Especially, the inclusion of book-value of equity seems to improve the forecasting models. More specifically, the results show that future earnings risk is positively related to the book-value of equity. As the number of options for further predictor variables is extremely large, future research may investigate other opportunities. Particularly the application of machine learning approaches might not only improve earnings variance forecasts, but additionally reveal important relationships between accounting-based financial data and future earnings variance.

Finally, this study finds that information about future earnings variance is priced in equity markets, but not considered in bond ratings. More specifically, equity prices are increasing in the variance of future earnings. Although this finding contradicts the model by Merton (1987), CH find the same relationship which is in line with the model by Pástor and Veronesi (2003). Additionally, CH hypothesize that the insignificant relationship between future earnings variance and bond ratings might be caused by rating agencies to be more concerned with past earnings variance compared to future earnings variance. A detailed investigation of this relationship is left to future research.

A Appendix

A.1 Variable Definitions

Panel A: Modelling the First Moment of Future Earnings

Variable	Description	COMPUSTAT Variable
$Earn$	Earnings divided by number of shares outstanding.	(IB-SPI)/CSHO
d^-	Indicator variable that equals 1 for firms with negative earnings and 0 otherwise.	
d^-Earn	Interaction term of $Earn$ and d^-.	
$BkEq$	Book value of equity divided by number of shares outstanding.	CEQ/CSHO
OCF	Cashflow divided by number of shares outstanding. XIDOC set to 0, if missing.	(OANCF-XIDOC)/CSHO
$TACC$	$Earn$ minus OCF.	

Panel B: Modelling The Second Moment of Future Earnings

Variable	Description	COMPUSTAT Variable
d^+	Indicator variable that equals 1 for firms with positive earnings and 0 otherwise.	
d^-TACC	Interaction term of $TACC$ and d^-.	
d^+TACC	Interaction term of $TACC$ and d^+.	
d^-OCF	Interaction term of OCF and d^-.	
d^+OCF	Interaction term of OCF and d^+.	
SPI	Special items divided by number of shares outstanding. Set to 0, if missing.	SPI/CSHO
d^-SPI	Interaction term of SPI and d^-.	
d^+SPI	Interaction term of SPI and d^+.	
LEV	Total assets divided by book-value of equity.	AT/CEQ
$PAYOUT$	Common dividends divided by shares outstanding.	DVPSX_F /CSHO
$PAYER$	Indicator variable that equals 1 for dividend payers and 0 otherwise.	
d^-BkEq	Interaction term of $BkEq$ and d^-.	

$d^{+}BkEq$	Interaction term of $BkEq$ and d^{+}.	
Industry Dummy	12 dummies which equal 1 if the firm belongs to the respective industry and 0 is not. Based on FF12.	

Panel C: Outcome and Control Variables

Variable	Description	COMPUSTAT Variable
$EP_{i,t}$	Earnings-to-price ratio of firm i in year t.	((IB-SPI)/CSHO)/PRC
$BR_{i,t}$	Long-term bond rating of firm i in year t divided into 21 categories from AAA equal to 1 up to D equal to 21.	SPLTICRM
$HflStd_{i,t+1}$	Forecasted firm-level standard deviation of earnings calculated as the standard deviation of firm i's realized earnings from year $t-9$ to t.	
$Size_{i,t}$	Equity market value of firm i in year t. Stock price data used from the CRSP monthly file.	PRC x CSHO
$Beta_{i,t}$	Market model beta of firm i in year t retrieved from WRDS Beta Suite.	
$AnnRet_{i,t}$	Firm i's annual stock return for year t. Calculation based on monthly returns of stock prices retrieved from the CRSP monthly file starting three months after fiscal-year end of year $t-1$.	
$RetStd_{i,t}$	Year t standard deviation of monthly market-model residuals for firm i resulting from regressing firm-level monthly returns on the respective market portfolio starting three months after fiscal-year end of year $t-1$. Prices and market portfolio returns are retrieved from CRSP.	PRC, VWRETD
$BP_{i,t}$	Book-to-market ratio of firm i in year t.	CEQ/(CSHO x PRC)
$LNSize_{i,t}$	The natural logarithm of the ratio of firm i's year t market-value of equity to the sum of all firm's market-values of equity in the respective year.	
$LiabAsset_{i,t}$	Ratio of liabilities to assets of firm i in year t.	LT/AT

$EbitdaLiab_{i,t}$	Ratio of EBITDA to liabilities of firm i in year t.	EBITDA/LT

A.2 Predictor Variable Sets

Table A9: Predictor Variable Sets for the Earnings Variance Forecast Models

	Predictor Variables
Set (I)	12 industry dummies, negative and positive earnings dummy, accruals interacted, cashflow interacted and special items interacted
Set (II)	Earnings, negative earnings dummy, earnings multiplied with the negative earnings dummy, accruals, leverage, dividends dummy and dividends
Set (III)	12 industry dummies, earnings, negative earnings dummy, earnings multiplied with the negative earnings dummy, accruals, leverage, dividends dummy and dividends
Set (IV)	12 industry dummies, negative and positive earnings dummy, accruals interacted, cashflow interacted, special items interacted and book-value of equity interacted
Set (V)	12 industry dummies, earnings, negative earnings dummy, earnings multiplied with the negative earnings dummy, accruals, leverage, dividends dummy, dividends and and book-value of equity

The term "interacted" implies that the respective variable is included twice in the model. It is multiplied with the negative earnings dummy as well as with the positive earnings dummy.

A.3 Empirical Results

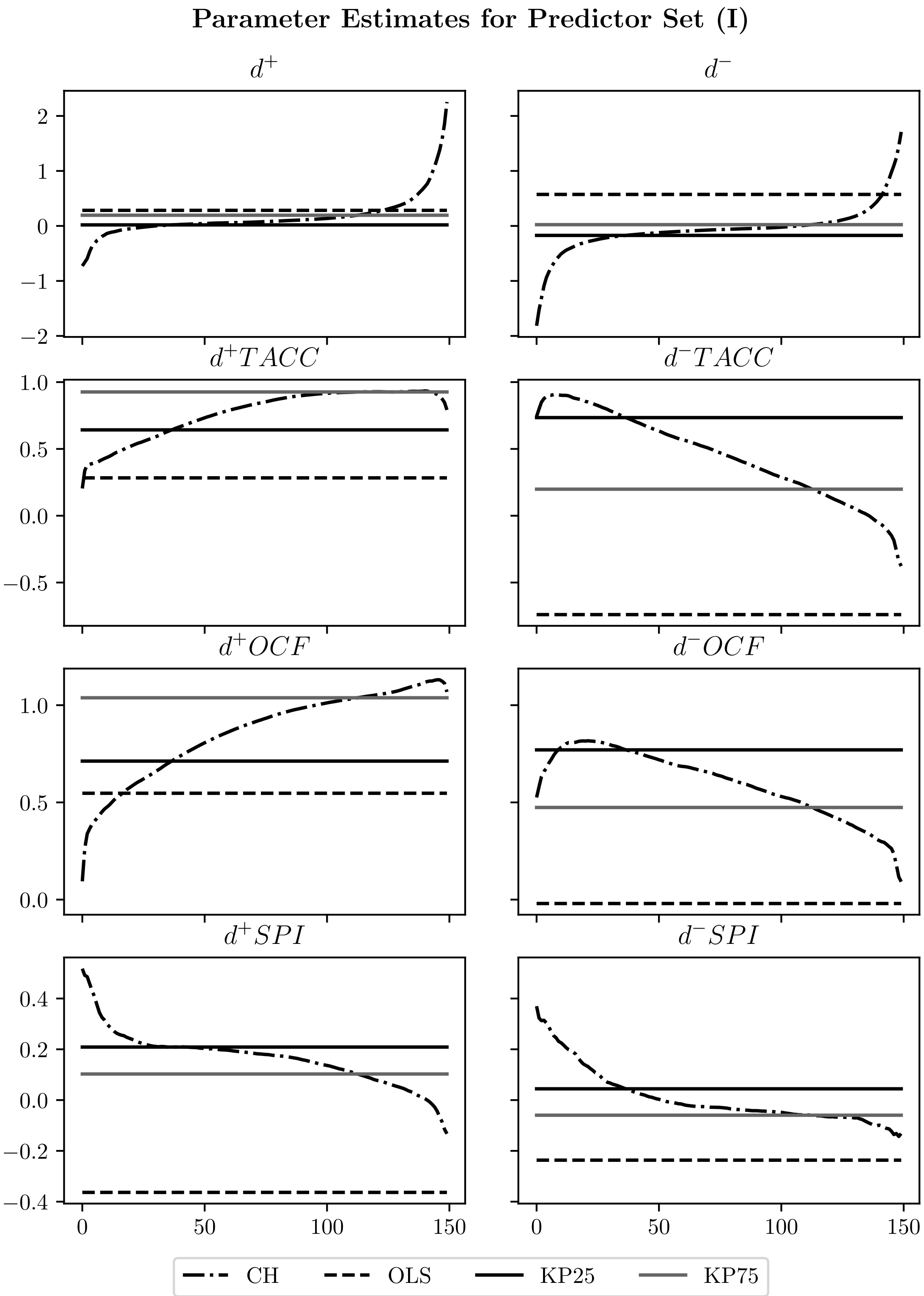
A.3.1 In-Sample Correlations

Table A10: Cross-Sectional Correlations

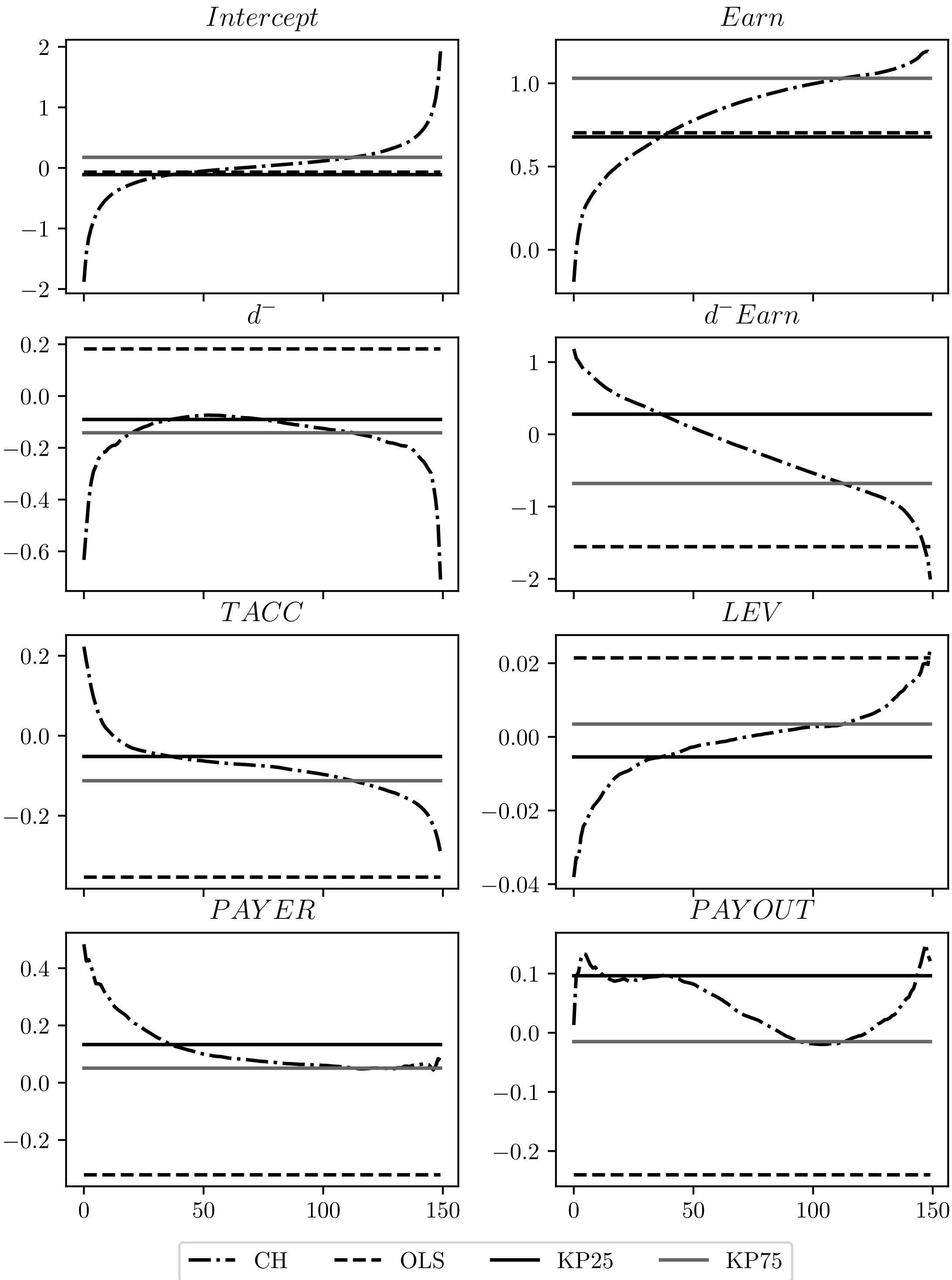
	$Earn$	d^+	d^-	$BkEq$	$TACC$	OCF	SPI	LEV	$PAYOUT$	$PAYER$
$Earn$	1.0***	0.55***	-0.55***	0.61***	-0.08***	0.72***	-0.1***	0.04***	0.47***	0.38***
d^+	0.55***	1.0***	-1.0***	0.33***	0.01*	0.37***	-0.02***	0.02***	0.25***	0.35***
d^-	-0.55***	-1.0***	1.0***	-0.33***	-0.01*	-0.37***	0.02***	-0.02***	-0.25***	-0.35***
$BkEq$	0.61***	0.33***	-0.33***	1.0***	-0.38***	0.67***	-0.1***	0.03***	0.44***	0.39***
$TACC$	-0.08***	0.01*	-0.01*	-0.38***	1.0***	-0.72***	0.06***	-0.09***	-0.25***	-0.18***
OCF	0.72***	0.37***	-0.37***	0.67***	-0.72***	1.0***	-0.11***	0.09***	0.49***	0.39***
SPI	-0.1***	-0.02***	0.02***	-0.1***	0.06***	-0.11***	1.0***	-0.05***	-0.06***	-0.04***
LEV	0.04***	0.02***	-0.02***	0.03***	-0.09***	0.09***	-0.05***	1.0***	0.07***	0.06***
$PAYOUT$	0.47***	0.25***	-0.25***	0.44***	-0.25***	0.49***	-0.06***	0.07***	1.0***	0.64***
$PAYER$	0.38***	0.35***	-0.35***	0.39***	-0.18***	0.39***	-0.04***	0.06***	0.64***	1.0***

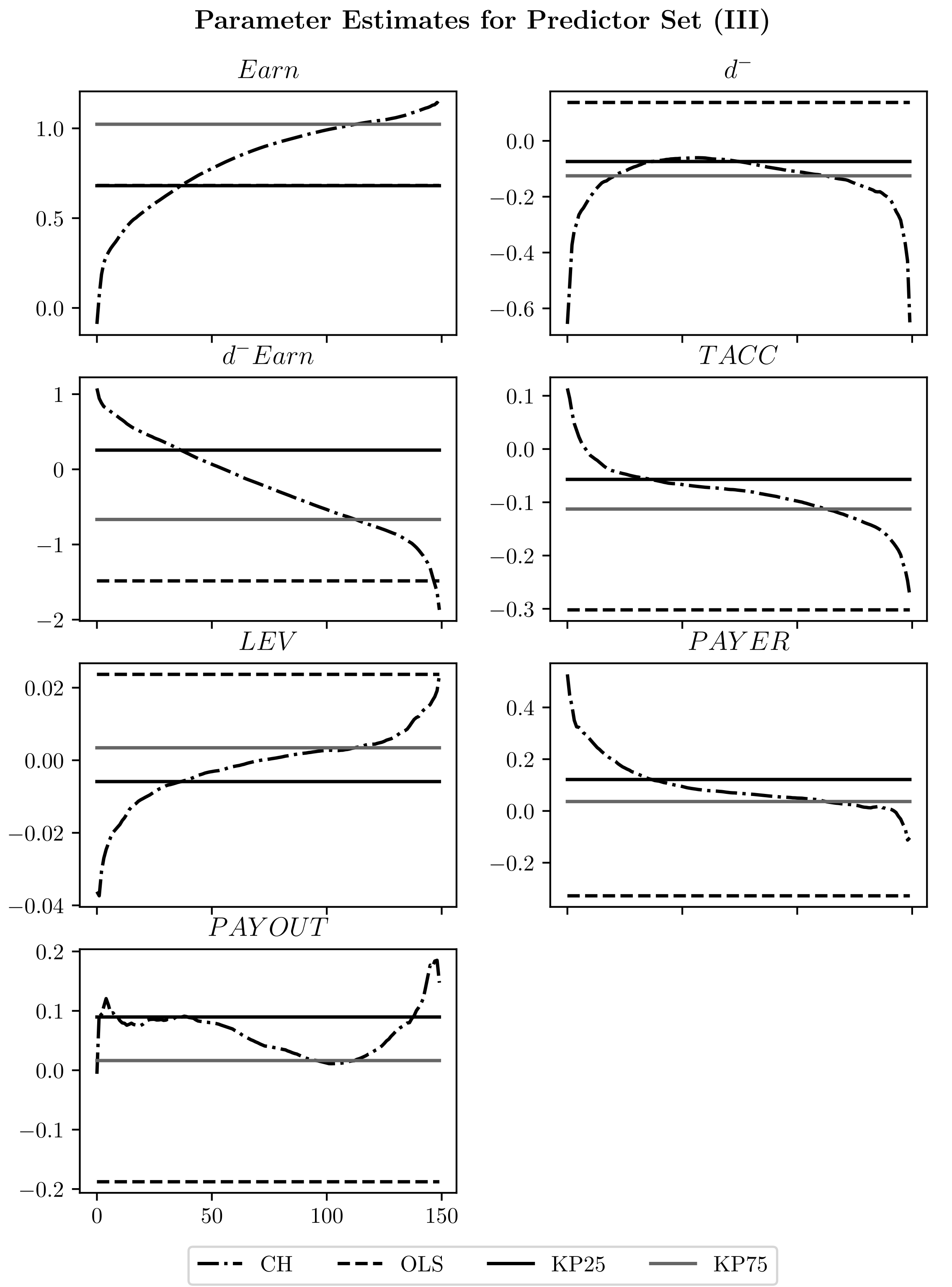
Table A10 displays Pearson correlations for the entire sample, i.e. the pooled cross-section of firms from 1988 to 2021 including all variables of the forecast models for the conditional first and second moment of future earnings. However, in order to conserve space, all interacted variables as well as the industry dummies are omitted in this table.

A.3.2 Parameter Estimates for the Variance Forecast Models



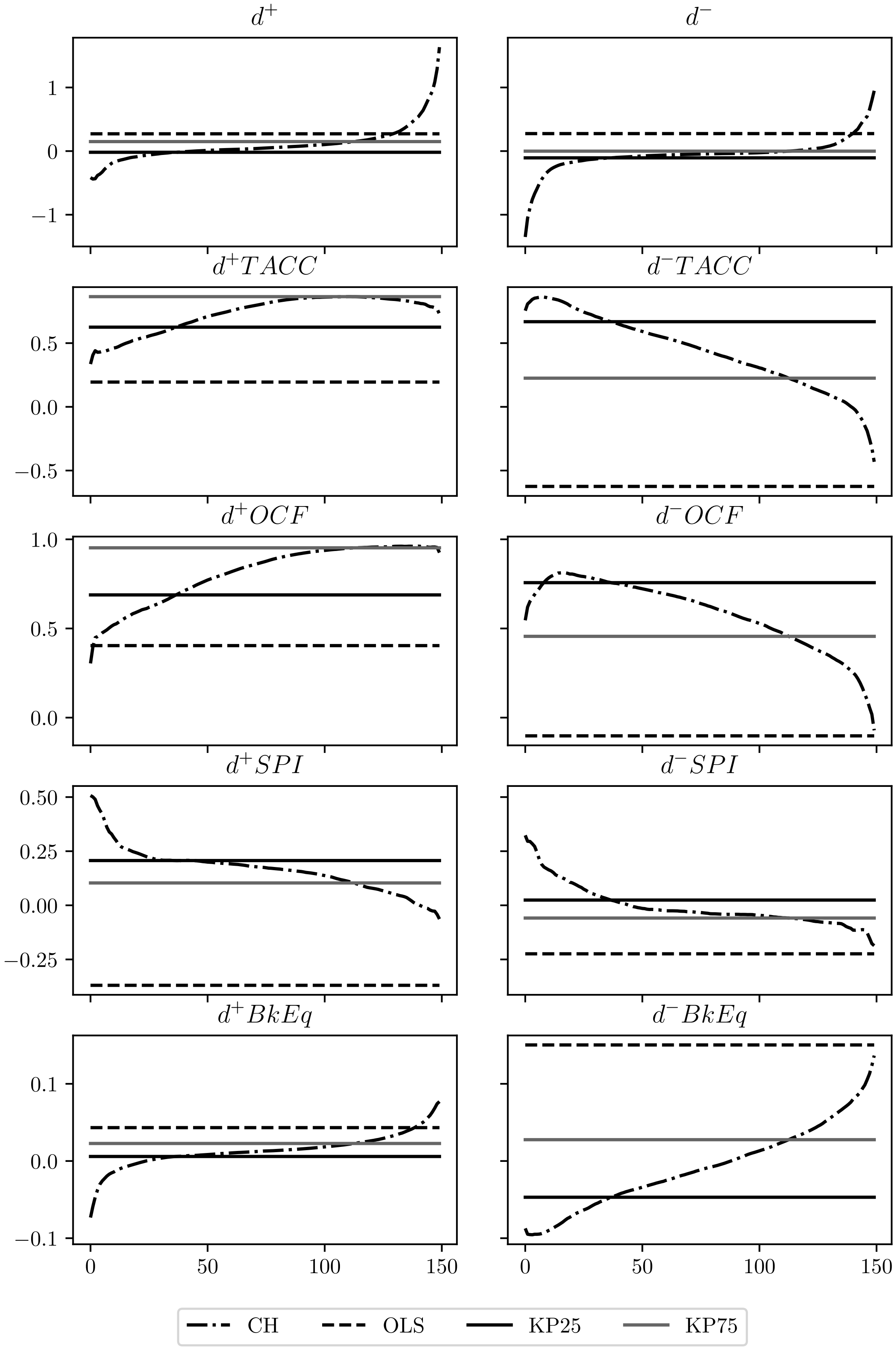
Parameter Estimates for Predictor Set (II)



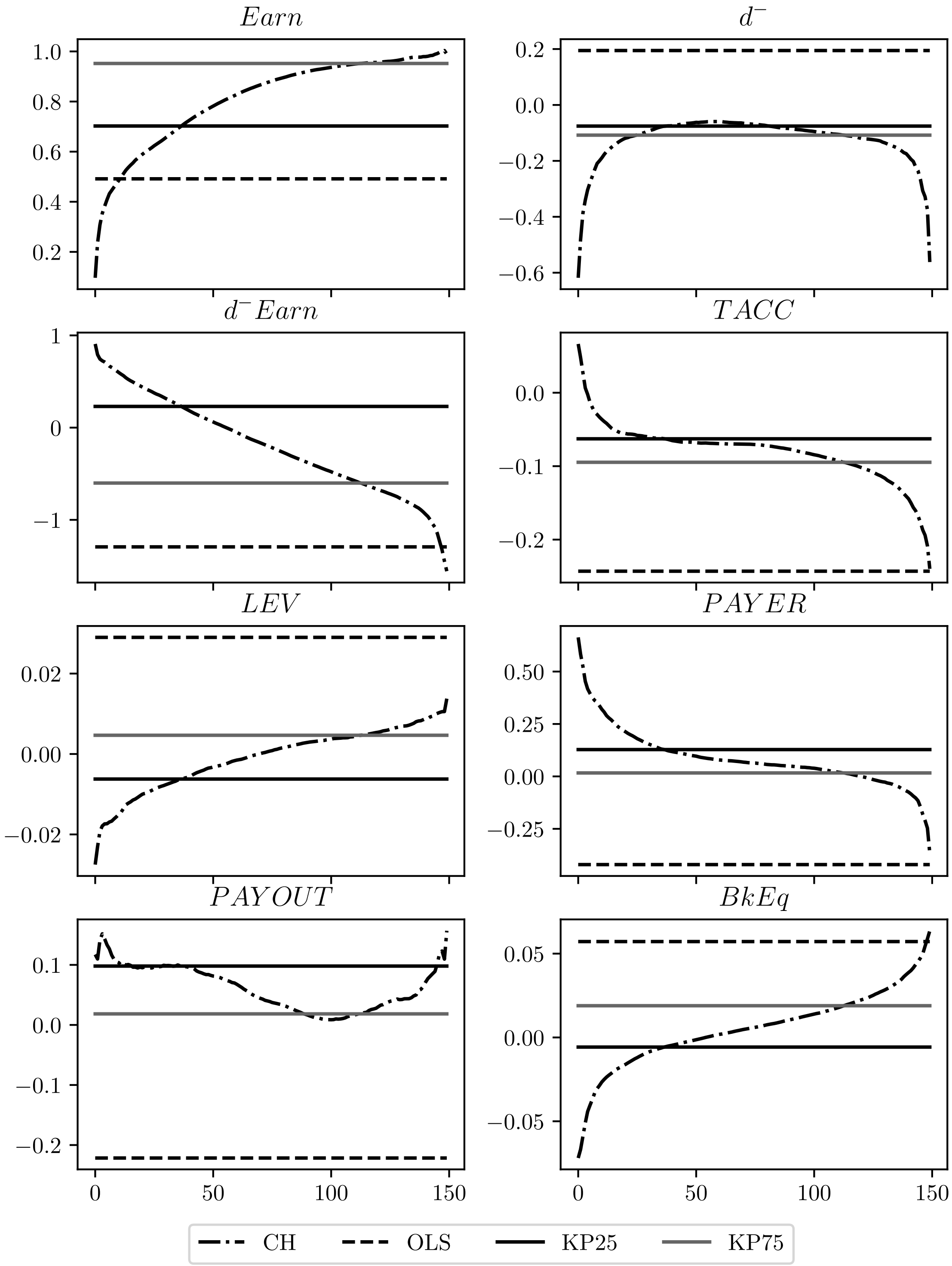


Parameter Estimates for Predictor Set (III)
Earn
d⁻
d⁻Earn
TACC
LEV
PAYER
PAYOUT
1.0
0.5
0.0
−0.2
−0.4
−0.6
1
0
−1
−2
0.1
−0.1
−0.3
0.02
0.00
−0.02
−0.04
0.4
0.2
0.2
0.1
0.0
−0.1
−0.2
0
50
100
150
CH
OLS
KP25
KP75

Parameter Estimates for Predictor Set (IV)



Parameter Estimates for Predictor Set (V)



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